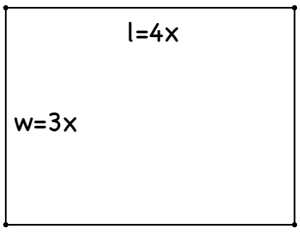
Seventh Annual Upper Peninsula High School Math Challenge

Northern Michigan University (Marquette, MI, USA)
Saturday 12 March 2016

Individual Problems — Solutions

1. The ratio of the width to the length of a rectangular rose garden is 3 to 4. If the perimeter of the garden is 168 feet, what is its area?

Answer: 1728 ft²



$14x = 168$ ft
$x = 12$
$w = 36$ ft
$l = 48$ ft
area $= 1728$ ft^2

2. In a sequence of consecutive integers the sum of the third and fourth terms is 47. What is the sum of the first five terms?

Answer: 115

$t_3 + t_4 = 47 \Rightarrow t_3 = 23,\ t_4 = 24$
$\therefore t_1 + t_2 + t_3 + t_4 + t_5 = 21 + 22 + 23 + 24 + 25 = 115$

3. The line $y = mx + m$ intersects the graph of $y = x^2$ at $x = m + 2$. Find the slope of the line.

Answer: slope = −4

$y = mx + m$ $\qquad$ $m\cdot(m + 2) + m = (m + 2)^2$
$y = x^2$ $\qquad$ $m^2 + 3m = m^2 + 4m + 4$
$x = m + 2$ $\qquad$ $-4 = m$

4. The sum of two numbers is 28, and their product is 7. Find the sum of the reciprocals of the numbers. Express the answer in simplest form.

Answer: 4

Let x and y be the two numbers. Then the sum of their reciprocals is

$$\frac{1}{x}+\frac{1}{y}=\frac{y+x}{xy}=\frac{28}{7}=4$$

5. A line with slope 2 intersects a line with slope 6 at the point (40, 30). What is the distance between the x-intercepts of these lines?

Answer: 10 units

$30 = 2(40) + b_1$

$30 = 6(40) + b_2$

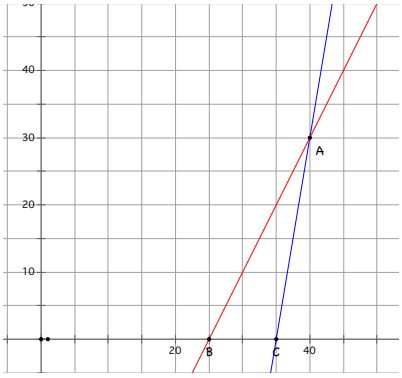
$b_1 = 30 - 80 = -50$

$b_2 = 30 - 240 = -210$

$0 = 2x_1 - 50 \Rightarrow x_1 = 25$

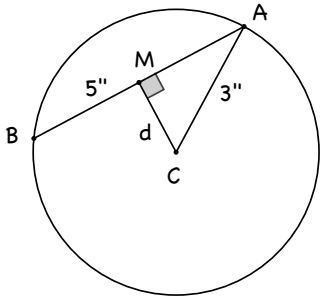
$0 = 6x_2 - 210 \Rightarrow x_2 = 35$

$x_2 - x_1 = 10$



6. Calculate the distance from the center of a circle of radius 3 inches to a chord of length 5 inches.

Answer: $d = \frac{\sqrt{11}}{2}$ inches



Distance from C to chord $\overline{AB}$ = d = $\overline{CM}$ where M is the midpoint of $\overline{AB}$. Also $\overline{CM} \perp \overline{AB}$ and $\overline{AM}$ = 2.5"

$$\therefore d^2 = 3^2 - \left(\frac{5}{2}\right)^2 = 9 - \frac{25}{4} = \frac{36-25}{4}$$

$$d = \sqrt{\frac{11}{4}} = \frac{\sqrt{11}}{2}$$

7. If $\frac{A}{B}+\frac{4}{3}+\frac{9}{2}=\frac{A}{B}\times\frac{4}{3}\times\frac{9}{2}$, find the value of $\frac{A}{B}$ in lowest terms.

Answer: $\frac{A}{B}=\frac{7}{6}$

$$\frac{A}{B}+\frac{4}{3}+\frac{9}{2}=\frac{A}{B}\times\frac{4}{3}\times\frac{9}{2}$$
$$\frac{A}{B}+\frac{8+27}{6}=\frac{A}{B}+\frac{35}{6}=\frac{A}{B}\times\frac{36}{6}$$
$$\frac{A}{B}+\frac{35}{6}=6\cdot\frac{A}{B}$$
$$\frac{35}{6}=5\cdot\frac{A}{B}$$
$$\frac{7}{6}=\frac{A}{B}$$

8. A bag contains only red and gold marbles. The probability of selecting a red marble is $\frac{2}{5}$, but if 20 red marbles are added to the bag, the probability of selecting a red marble becomes $\frac{4}{7}$. How many gold marbles are in the bag?

Answer: 30 gold marbles

Initial condition: $P(red)=\frac{2}{5}=\frac{2x}{5x}$

After adding 20 red: $P(red)=\frac{4}{7}=\frac{4y}{7y}$

red marbles: $2x+20=4y$

gold marbles: $3x=3y\Rightarrow x=y$

$\therefore 2x+20=4x\Rightarrow x=10$

$\therefore$ gold marbles: 30

9. How many distinct 3-digit numbers have a digit sum of 8?

Answer: 36

How many ways can we get a sum of 8?	Count Permutations	
1 + 1 + 6 = 8	116 161 611	3
1 + 2 + 5 = 8	125 152 215 251 512 521	6
1 + 3 + 4 = 8	134 143 314 341 413 431	6
2 + 2 + 4 = 8	224 242 422	3
2 + 3 + 3 = 8	233 323 332	3
0 + 1 + 7 = 8	107 170 701 710	4
0 + 2 + 6 = 8	260 206 620 602	4
0 + 3 + 5 = 8	350 305 530 503	4
0 + 4 + 4 = 8	440 404	2
0 + 0 + 8 = 8	800	1
Total number:		36

10. If $2^4 \cdot 4^8 \cdot 8^{16} \cdot 16^{32} = 32^X$, what is x?

Answer: 39.2

$$2^4 \cdot 4^8 \cdot 8^{16} \cdot 16^{32} = 2^4 \cdot 2^{16} \cdot 2^{48} \cdot 2^{128} = 32^x = 2^{5x}$$
$$5x = 4 + 16 + 48 + 128 = 196$$
$$x = 39.2$$