

Fifth Annual Upper Peninsula
High School Math Challenge
Northern Michigan University
(Marquette Co, MI)
Saturday 15 March 2014

SCHOOL: SOLUTION

TEAM: _____

RELAY: 1

1. 75

2. 48

3. 17

4. 9

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SCHOOL: SOLUTION

TEAM: _____

RELAY: 2

1. $\frac{3}{9}$

2. 110

3. 9

4. 130

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SCHOOL: SOLUTION

TEAM: _____

RELAY: 3

1. 1

2. $\sqrt{5}$

3. $\frac{5\sqrt{3}}{3}$

4. 8

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SCHOOL: _____

TEAM: _____

RELAY: _____

1. _____

2. _____

3. _____

4. _____

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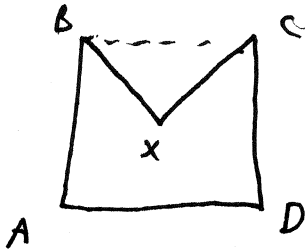
RELAY 1

Category: GEOMETRY

PLAYER 1

Square ABCD has center X. If $AB = 10$, compute the area of pentagon AXBCD.

Pass your answer to Player 2.



Area of square is $10^2 = 100$
Area of pentagon is $\frac{3}{4} \cdot 100 = 75$

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RELAY 1

Category: RATE TIME DISTANCE

PLAYER 2

The number you will receive from Player 1 is the speed in miles per hour that Tom is driving from City A to City B. Jane is driving at $\frac{2}{3}$ of that speed from City B to City A. If the cities are 100 miles apart, and Tom and Jane start driving at the same time, how long in minutes, after they start driving, will they pass each other?

Pass your answer to Player 3.

A •

• B

A's speed is 75

B's speed is 50

$$75t + 50t = 100$$

$$125t = 100$$

$$t = \frac{100}{125} = \frac{4}{5} \text{ hr} = \frac{4}{5} \cdot 60 = 48 \text{ min.}$$

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RELAY 1

Category: GREGORIAN CALENDAR

PLAYER 3

Add 1900 to the number you will receive from Player 2, and interpret this number as a year. How many leap years are there between that year and the current year (inclusively)?

Pass your answer to Player 4.

- 1948
- 1952
- 1956
- 1960
- 1964
- 1968
- 1972
- 1976
- 1980
- 1984
- 1988
- 1992
- 1996
- 2000
- 2004
- 2008
- 2012

17

$$\text{or } \frac{2012 - 1948}{4} + 1$$
$$\frac{64}{4} + 1$$
$$17$$

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RELAY 1

PLAYER 4

Category: AGE PROBLEM

The number you will receive from Player 3 is my current age in years. When my sister is two years older than I am now, I will be three times her present age. How old is my sister now?

Run your answer to the front.

$s =$ sister's age

I am 17.

$$17 + \overbrace{19-s}^{\text{years in future}} = 3s$$

$$36 - s = 3s$$

$$4s = 36$$

$$s = 9$$

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RELAY 2

Category: PROBABILITY

PLAYER 1

Given two standard dice, what is the probability of rolling a 7 or 11?

Pass your answer to Player 2.

$$7 \Rightarrow 1, 6$$

$$2, 5$$

$$3, 4$$

$$4, 3$$

$$5, 2$$

$$6, 1$$

$$11 \Rightarrow 5, 6$$

$$6, 5$$

8 ways out of 36 possibilities

$$\frac{8}{36} = \frac{2}{9}$$

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RELAY 2

Category: PERMUTATIONS AND COMBINATIONS

PLAYER 2

The number you will receive from Player 1 is a fraction. Put the fraction into reduced form (if necessary) and let S be the sum of the numerator and denominator. Suppose you had a bag containing S balls, one blue, one green, and the rest red. Assuming the red balls cannot be distinguished from each other, how many ways are there to arrange the balls in a row?

Pass your answer to Player 3.

$$S = 11$$

one blue ball one green ball nine red balls

The permutations are $\frac{11!}{1!1!9!} = \frac{11 \cdot 10 \cdot 9!}{9!} = 110$

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RELAY 2

Category: PALINDROMES

PLAYER 3

The number you will receive from Player 2 is a positive integer. What is the *smallest* positive integer you can subtract from that integer to yield a palindrome (a number reading the same forwards and backwards)?

Pass your answer to Player 4.

From 110, I can subtract 9 to reach 101.

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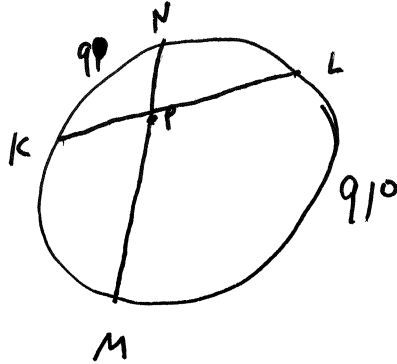
RELAY 2

Category: CIRCLES

PLAYER 4

P is a point in the interior of circle C. Chords KL and MN intersect at P; $\angle LPM$ subtends an arc of 91° ; $\angle KPN$ subtends an arc whose measure in degrees is the number you will receive from Player 3. What is the measure of $\angle KPM$ in degrees?

Run your answer to the front.



$$\angle KPN = \angle LPM = \frac{91 + 91}{2} = 91$$

$$\angle KPM = 180^\circ - \angle KPN = 180^\circ - 91^\circ = 89^\circ$$

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RELAY 3

Category: SIMULTANEOUS EQUATIONS

PLAYER 1

If $x + y + z = 3$ and $2x + 2y = z$, what is the sum of x and y ?

Pass your answer to Player 2.

$$\cancel{2x + 2y + z = 6}$$

$$2x + 2y - z = 0$$

$$- \textcircled{x}$$

$$x + y + z = 3$$

$$3x + 3y = 3$$

$$x + y = 1$$

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RELAY 3

PLAYER 2

Category: SUM, PRODUCT, AND DIFFERENCE OF ROOTS

The number you will receive from Player 1 is both the absolute value of the difference between the roots of a quadratic equation and the product of the roots of that same equation. What is the absolute value of the sum of the roots of this equation?

Pass your answer to Player 3.

~~$$r_1 - r_2 = 1$$

$$r_1 r_2 = 1$$

$$(1 + r_2) r_2 = 1$$

$$r_2 + r_2^2 = 1$$~~

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$r_1 - r_2 = \frac{2\sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{b^2 - 4ac}}{a}$$

$$r_1 + r_2 = -\frac{b}{a}$$

$$r_1 r_2 = \frac{c}{a}$$

If we set a (the leading coefficient) to 1, we have

$$1 = r_1 r_2 = c$$

$$1 = r_1 - r_2 = \sqrt{b^2 - 4c} = \sqrt{b^2 - 4}$$

$$\sqrt{b^2 - 4} = 1$$

$$b^2 - 4 = 1$$

$$b^2 = 5$$

$$b = \pm\sqrt{5}$$

$$r_1 + r_2 = -b = \pm\sqrt{5}$$

$$|r_1 + r_2| = \sqrt{5}$$

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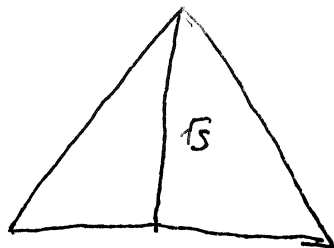
RELAY 3

Category: TRIANGLES

PLAYER 3

The number you will receive is the height of an equilateral triangle. What is the area of this triangle?

Pass your answer to Player 4.



$$h = \frac{s\sqrt{3}}{2}$$

$$15 = \frac{s\sqrt{3}}{2}$$

$$s = \frac{2\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{15}}{3}$$

$$A = \frac{s^2\sqrt{3}}{4}$$

$$= \frac{\frac{4 \cdot 15}{9} \sqrt{3}}{4} = \frac{15\sqrt{3}}{9} = \frac{5\sqrt{3}}{3}$$

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RELAY 3

Category: COIN PROBLEM

PLAYER 4

Square the number you will receive from Player 3 and express it as a reduced (improper) fraction. The denominator of this fraction is the number of coins you have. The numerator is how much each coin is worth (in cents). Your friend Steve has the same amount of money that you have but all of his coins are worth less than your coins. Assuming Steve's coins are U.S. currency, what is the minimum number of coins that Steve could have?

Run your answer to the front.

$$\left(\frac{5\sqrt{3}}{3}\right)^2 = \frac{75}{9} = \frac{25}{3}$$

I have three quarters, or 75¢
without quarters, Steve has 7 dimes and 1 nickel,
or 8 coins,