

*Fifth Annual Upper Peninsula  
High School Math Challenge  
Northern Michigan University (Marquette Co, MI)  
Saturday 15 March 2014*

NAME: SOLUTION

TEAM: \_\_\_\_\_

SCHOOL: \_\_\_\_\_

**PROBLEM 1**

**TIME: 4 minutes**

$$\frac{1}{4}$$

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**answer**

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Evaluate

$$\frac{(2 + \sqrt{2})^2 - 4\sqrt{2}}{(\sqrt[3]{3} + \sqrt[3]{3})^3}$$

Express your answer as a reduced fraction with a rationalized denominator.

$$\begin{aligned} \frac{(2 + \sqrt{2})^2 - 4\sqrt{2}}{(\sqrt[3]{3} + \sqrt[3]{3})^3} &= \frac{4 + 4\sqrt{2} + 2 - 4\sqrt{2}}{(2\sqrt[3]{3})^3} \\ &= \frac{6}{8 \cdot 3} = \frac{6}{24} = \frac{1}{4} \end{aligned}$$

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**PROBLEM 2**

**TIME: 3 minutes**

$12\frac{1}{2}$  mi/hr

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answer

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Rob takes the same amount of time to walk 3 miles as it takes Tom to ride his bike 5 miles. If Tom travels 5 miles per hour faster than Rob, what is Tom's speed, in miles per hour?

$$r = \text{Rob's speed (mi/hr)}$$

$$\textcircled{+} r + 5 = \text{Tom's speed (mi/hr)}$$

$$t = \text{common time}$$

$$r = \frac{3}{t}$$

$$t = \frac{3}{r}$$

$$r + 5 = \frac{5}{t}$$

$$t = \frac{5}{r + 5}$$

$$\frac{3}{r} = \frac{5}{r + 5}$$

$$3(r + 5) = 5r$$

$$3r + 15 = 5r$$

$$2r = 15$$

$$r = \frac{15}{2} = 7\frac{1}{2}$$

$$r + 5 = 12\frac{1}{2} \text{ mi/hr}$$

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**PROBLEM 3**

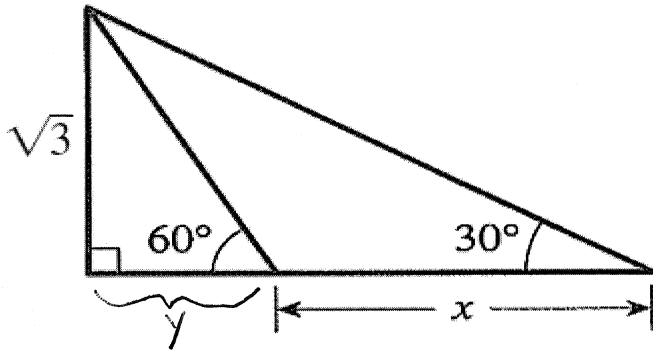
**TIME: 4 minutes**

2

answer

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Find x



$$\tan 60^\circ = \sqrt{3}$$

$$\frac{\sqrt{3}}{y} = \sqrt{3}$$

$$\frac{1}{y} = 1$$

$$y = 1$$

$$\tan 30^\circ = \frac{\sqrt{3}}{3}$$

$$\frac{\sqrt{3}}{x+1} = \frac{\sqrt{3}}{3}$$

$$x+1=3$$

$$x=2$$

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**PROBLEM 4**

**TIME: 3 minutes**

4536

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answer

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

How many integers between 1000 and 9999 have distinct digits? In other words, how many of them have no repeated digits?

The first digit can have 9 possibilities,

The second digit also has 9 possibilities,

The third digit has 8 possibilities,

The fourth digit has 7 possibilities.

$$9 \cdot 9 \cdot 8 \cdot 7 = 4536$$



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**PROBLEM 5**

**TIME: 4 minutes**

90

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answer

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Imagine the product of  $20^{50}$  and  $50^{20}$  written as an integer in standard form. How many zeros will be found at the end of this number?

$$20^{50} \cdot 50^{20}$$

$$2^{50} \cdot 10^{50} \cdot 5^{20} \cdot 10^{20}$$

$$2^{50} \cdot 5^{20} \cdot 10^{70}$$

$$2^{30} \cdot 2^{20} \cdot 5^{20} \cdot 10^{70}$$

$$2^{50} \cdot 10^{20} \cdot 10^{70}$$

$$2^{50} \cdot 10^{90}$$

So, 90 zeros will be at the end

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**PROBLEM 6**

**TIME: 5 minutes**

25

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answer

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Find all positive values of  $x$  that satisfy the following equation:

$$\log x + \log(x - 21) = 2$$

~~$$\log [x(x-21)] = 2$$
  
$$x(x-21) = 10^2$$
  
$$x^2 - 21x - 100 = 0$$~~

$$\log [x(x-21)] = 2$$

$$x(x-21) = 10^2$$

$$x^2 - 21x - 100 = 0$$

$$(x-25)(x+4) = 0$$

$$x = 25$$

$$x = -4$$

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**PROBLEM 7**

**TIME: 5 minutes**

$1 - \sqrt{2}, 1, 1 + \sqrt{2}$

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answer

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Find all real values of  $x$  that satisfy the following equation:

$$|x^2 - 2x| = 1$$

$$x^2 - 2x = 1$$

$$\text{OR } x^2 - 2x = -1$$

$$x^2 - 2x - 1 = 0$$

$$x^2 - 2x + 1 = 0$$

$$x = \frac{2 \pm \sqrt{4 + 4}}{2}$$

$$(x-1)^2 = 0$$

$$x = 1$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= 1 \pm \sqrt{2}$$

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**PROBLEM 8**

**TIME: 5 minutes**

$(-2, 3)$

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answer

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

Find all real  $(x, y)$  pairs that satisfy the following equation:

$$x^2 + y^2 + 4x + 13 = 6y$$

$$x^2 + 4x + y^2 - 6y + 13 = 0$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 0$$

$$(x+2)^2 + (y-3)^2 = 0$$

$$(x+2)^2 = 0$$

$$(y-3)^2 = 0$$

~~2~~

$$x = -2$$

$$y = 3$$



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**PROBLEM 9**

**TIME: 3 minutes**

7

answer

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

A polygon has twice as many diagonals as it has sides. How many sides does it have?

Every vertex has a diagonal to every other vertex EXCEPT itself and its two adjacent vertices,

A polygon with  $n$  sides has  $\frac{n(n-3)}{2}$  <sup>diagonals</sup> ~~vertices~~, (since each diagonal was counted twice).

$$\frac{n(n-3)}{2} = 2n$$

$$n^2 - 3n = 4n$$

$$n^2 - 7n = 0$$

$$n(n-7) = 0$$

$$n = 0 \quad n = 7$$

↑

doesn't  
work

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**PROBLEM 10**

**TIME: 4 minutes**

11

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answer

**Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.**

We are given three consecutive integers. The difference between the cubes of the two larger of the three consecutive integers is 66 more than the difference between the cubes of the two smaller integers. What is the median integer?

The integers are

$$x-1 \quad x \quad x+1$$

$$(x+1)^3 - x^3 = 66 + x^3 - (x-1)^3$$

$$x^3 + 3x^2 + 3x + 1 - x^3 = 66 + x^3 - x^3 + 3x^2 - 3x + 1$$

$$3x^2 + 3x + 1 = 3x^2 - 3x + 67$$

$$3x + 1 = -3x + 67$$

$$6x = 66$$

$$x = 11$$