Fourth Annual Upper Peninsula
High School Math Challenge
Northern Michigan University
(Marquette Co, MI)
Saturday 23 March 2013

SCHOOL: SOLUTION

TEAM: _____

RELAY: 1

1. $45\sqrt{2}$
2. 15
3. 156
4. 48

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SCHOOL: SOLUTION

TEAM: _____

RELAY: 2

1. 202
2. 30
3. 7
4. $\frac{1}{6}$

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SCHOOL: SOLUTION

TEAM: _____

RELAY: 3

1. $\frac{1}{6}$
2. 120
3. $5\sqrt{3}$
4. 8

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SCHOOL: _____

TEAM: _____

RELAY: _____

1. _____
2. _____
3. _____
4. _____

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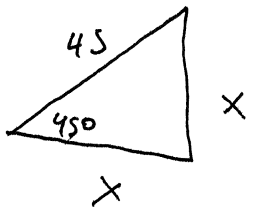
RELAY 1 **PLAYER 1**

Category: RIGHT TRIANGLES

I walk forty-five miles up a hill with a 45° incline. When I reach the top, what is my vertical distance, in miles, from the plain?

Pass your answer to Player 2.



$$x^2 + x^2 = 45^2$$

$$2x^2 = 45^2$$

$$x^2 = \frac{45^2}{2}$$

$$x = \frac{45}{\sqrt{2}}$$

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RELAY 1 **PLAYER 2**

Category: RATE TIME DISTANCE

Multiply the number you receive from Player 1 by $\sqrt{2}$. This is the number of minutes that you have been driving at 40 MPH. Your friend started at the same time and place as you. He is driving in the same direction but at half your speed. In miles, how far apart are the two of you now?

Pass your answer to Player 3.

$\frac{45}{\sqrt{2}} \cdot \sqrt{2} = 45$

I am moving away from him at 20 MPH. We have been driving for $\frac{3}{4}$ hr.

$\frac{3}{4} \cdot 20 = 15$

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RELAY 1 **PLAYER 3**

Category: POLYGONS

The number you receive from Player 2 is the number of sides of a regular polygon. What is the measure, in degrees, of each angle of that polygon?

Pass your answer to Player 4.

$$\frac{n-2}{n} \cdot 180$$

$$\frac{13}{15} \cdot 180 = 13 \cdot 12 = 156$$

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RELAY 1 **PLAYER 4**

Category: RATIOS

I have four times as many pepperoni pizzas as cheese pizzas. I have eight times as many sausage pizzas as cheese pizzas. The number you receive from Player 3 is the total number of pizzas I have. How many of them are pepperoni pizzas?

Run your answer to the front.

$p = 4c$

$s = 8c$

$4c + 8c + c = 156$

$13c = 156$

$c = 12$

$p = 4c = 48$

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RELAY 2 **PLAYER 1**

Category: DIGIT PROBLEM

The sum of the digits of an even three-digit number is 4. This number is also a palindrome (reading the same backwards as forwards). What is this number?

Pass your answer to Player 2.

only 3 digit palindromes with a digit sum of 4 are 202 and 121. Only 202 is even.

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RELAY 2 **PLAYER 2**

Category: INFINITE SERIES

Let n be the one's digit of the number you receive from Player 1. A ball is dropped from a height of ten feet; each time it bounces, the ball rises to $1/n$ of its previous height. In feet, how far does the ball travel before coming to rest?

Pass your answer to Player 3.

$n=2$ Ball falls: $10 + \frac{1}{2}10 + \frac{1}{4}10 + \ldots = \frac{10}{(1-\frac{1}{2})} = \frac{10}{\frac{1}{2}} = 20$

Ball rises: $5 + \frac{1}{2}\cdot 5 + \frac{1}{4}\cdot 5 + \ldots = \frac{5}{1-\frac{1}{2}} = 5/\frac{1}{2} = 10$

Ball travels 30 ft.

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RELAY 2 **PLAYER 3**

Category: INTEGER PROGRAMMING

The number you receive from Player 2 is the amount of money in dollars that I spend on comic books. If Batman comics cost four dollars apiece and Spider-Man comics cost five dollars apiece, what is the *largest* number of comics I can buy without having any money left over?

Pass your answer to Player 4.

$4b + 5s = 30$

$b = 0 \quad s = \frac{30}{5} = 6$

$b = 1 \quad s = \frac{26}{5}$

$b = 2 \quad s = \frac{22}{5}$

$b = 3 \quad s = \frac{18}{5}$

$b = 4 \quad s = \frac{14}{5}$

$b = 5 \quad s = \frac{10}{5} = 2$ 7 comics

$b = 6 \quad s = \frac{6}{5}$

$b = 7 \quad s = 2/5$

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RELAY 2 **PLAYER 4**

Category: PROBABILITY

I have two regular dice each numbered 1 - 6. The number you receive from Player 3 is the number I get when I throw both dice. Assuming the dice are fair, what is the probability of this happening?

Run your answer to the front.

7 : 1+6
: 2+5
: 3+4
: 4+3
: 5+2
: 6+1

$\frac{6}{36} = \frac{1}{6}$

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RELAY 3 **PLAYER 1**

Category: QUADRATIC EQUATIONS

What is the absolute value of the difference of the roots of the equation

$6x^2 - 5x + 1 = 0$?

Pass your answer to Player 2.

Difference given by $\frac{\sqrt{b^2-4ac}}{a} = \frac{\sqrt{25-24}}{6} = \frac{1}{6}$

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RELAY 3 **PLAYER 2**

Category: COUNTING PROBLEM

The number you receive from Player 1 is the fraction of a group of 24 people that are Canadian. The rest are American. How many ways are there to draw three people from that group with two being Canadian and one being American?

Pass your answer to Player 3.

$\frac{1}{6} \cdot 24 = 4$ Canadians

20 Americans

$4C_2 \cdot 20C_1$

$6 \cdot 20 = 120$

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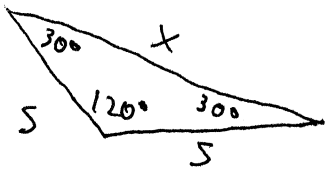
RELAY 3 **PLAYER 3**

Category: TRIGONOMETRY

The number you receive from Player 2 is the measure, in degrees, of one angle of a triangle. The sides adjacent to that angle both have length 5. What is the length of the third side?

Pass your answer to Player 4.



$$\frac{x}{\sin 120^\circ} = \frac{5}{\sin 30^\circ}$$

$$5 \sin 120^\circ = x \sin 30^\circ$$

$$5 \frac{\sqrt{3}}{2} = x \frac{1}{2}$$

$$x = 5\sqrt{3}$$

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RELAY 3 **PLAYER 4**

Category: COIN PROBLEM

Let *n* be the *square* of the number you will receive from Player 3.

Using only nickels and/or dimes, how many ways are there to make exactly *n* cents?

Run your answer to the front.

75 cents

0 d 15 n

1 d 13 n

2 d 11 n

3 d 9 n

4 d 7 n

5 d 5 n

6 d 3 n

7 d 1 n

8 ways