

*Fourth Annual Upper Peninsula
High School Math Challenge
Northern Michigan University (Marquette Co, MI)
Saturday 23 March 2013*

NAME: SOLUTION

TEAM: _____

SCHOOL: _____

PROBLEM 1

TIME: 3 minutes

50°

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

The angles of a triangle have measures of $(x + 20)^\circ$, $(2x + 15)^\circ$, and $(2x - 5)^\circ$. Find, in degrees, the measure of the *smallest* angle of this triangle.

$$x + 20 + 2x + 15 + 2x - 5 = 180$$

$$5x + 30 = 180$$

$$5x = 150$$

$$x = 30$$

Angles are

$$30^\circ, 75^\circ, 55^\circ$$

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PROBLEM 2

TIME: 3 minutes

75 & 13

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

The sum of two integers is 88. If the larger integer is divided by the smaller integer, the quotient is 5 and the remainder is 10. What are the two integers?

$$l + s = 88$$

$$\frac{l}{s} = 5 + \frac{10}{s}$$

$$l = 88 - s$$

$$l = 5s + 10$$

$$88 - s = 5s + 10$$

$$78 = 6s$$

$$s = 13$$

$$l = 88 - 13 = 75$$

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PROBLEM 3

TIME: 3 minutes

TS/PA/VA 35

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

Seven points lie on a circle. How many inscribed triangles can be constructed having these points as vertices?

How many ways are there to choose
3 points out of 7?

$${}^7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3!4!} = \frac{7 \times 6 \times 5}{6} = 35$$

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PROBLEM 4

TIME: 4 minutes

-3, 9, 10

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

Find three numbers such that when added pairwise (two at a time) the sums are 19, 6, and 7.

$$x + y = 19$$

$$x + z = 6$$

$$x = 9$$

$$x + z = 6$$

$$9 + z = 6$$

$$z = -3$$

$$y + z = 7$$

$$-x - z = -6$$

$$y - x = 1$$

$$y + x = 19$$

$$2y = 20$$

$$y = 10$$

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PROBLEM 5

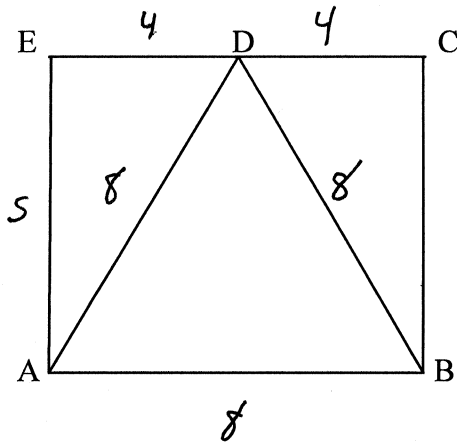
TIME: 4 minutes

$$4\sqrt{3}$$

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

Equilateral triangle ABD is inscribed in rectangle ABCE. $BD = 8$. Find AE.



$$s^2 + 4^2 = 8^2$$

$$s^2 + 16 = 64$$

$$s^2 = 48$$

$$s = \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$$

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PROBLEM 6

TIME: 4 minutes

3

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

Find the radius of the circle whose equation is:

$$x^2 + y^2 - 4x + 2y = 4$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 = 4 + 4 + 1$$

$$(x-2)^2 + (y+1)^2 = 9$$

$$r = \sqrt{9} = 3$$

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PROBLEM 7

TIME: 5 minutes

ln 3, ln 4

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

Find all real values of x that satisfy the following:

$$e^{2x} - 7e^x + 12 = 0$$

$$\text{Let } y = e^x$$

$$y^2 - 7y + 12 = 0$$

$$(y-3)(y-4) = 0$$

$$y-3=0 \quad y-4=0$$

$$y=3 \quad y=4$$

$$e^x=3 \quad e^x=4$$

$$x = \ln 3 \quad x = \ln 4$$

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PROBLEM 8

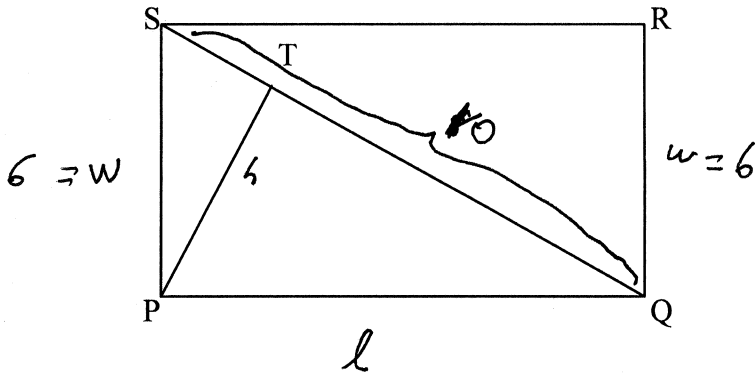
TIME: 5 minutes

4.8

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

In rectangle PQRS, $QR = 6$, $SQ = 10$, $\overline{PT} \perp \overline{SQ}$. Find PT.



$$l^2 + 6^2 = 10^2$$

$$l^2 + 36 = 100$$

$$l^2 = 64$$

$$l = 8$$

$$A = \frac{1}{2}bh = \frac{1}{2}lw$$

$$bh = lw$$

$$10h = 8 \cdot 6$$

$$10h = 48$$

$$h = 4.8$$

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PROBLEM 9

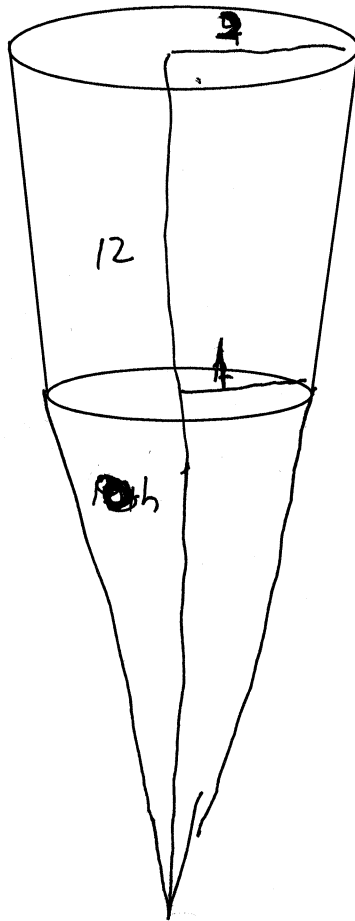
TIME: 5 minutes

28π

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

At the movie theater, I get the Big Gulp drink. The sides of the cup have a uniform slope. The height of the cup is 12 inches. The top of the cup is 4 inches across. The bottom of the cup is 2 inches across. If I fill the cup to the brim with Coke, how much Coke is in the cup, in cubic inches?



$$\frac{1}{h} = \frac{2}{12+h}$$

(similar triangles)

$$2h = 12+h$$

$$h = 12$$

Volume of small cone is $\frac{1}{3} \pi 1^2 \cdot 12 = 4\pi$

Volume of large cone is $\frac{1}{3} \pi 2^2 \cdot 24 = 32\pi$

Volume of cup is $32\pi - 4\pi = 28\pi$

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PROBLEM 10

TIME: 5 minutes

249

answer

Put no work on this side of the paper. Write the answer and only the answer in the space above. Put all work on the other side of the sheet.

If $1000!$ were written out completely (in base ten), how many zeroes would be at the end?

What is the largest ~~factor~~ k such that $1000!$ is divisible by 10^k ?

$10^k = 2^k \cdot 5^k$ $1000!$ will contain more factors of 2 than 5, so it suffices to count the factors of 5,

$$1000! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot 1000$$

$$\frac{1000}{5} = 200 \text{ are divisible by } 5$$

$$\frac{200}{5} = 40 \text{ are divisible by } 5^2$$

$$\frac{40}{5} = 8 \text{ are divisible by } 5^3$$

$$\frac{8}{5} = 1 \text{ are divisible by } 5^4$$

$200 + 40 + 8 + 1 = 249$ factors of 5, so 249 factors of 10, so 249 ~~factors~~ zeroes at the end.