

Problem 1—Egyptian Multiplication

In 1858, A. Henry Rhind, a Scottish antiquary, came into possession of a document which is now called the Rhind Papyrus. Titled “Directions for Attaining Knowledge into All Obscure Secrets,” the document provides important clues as to how the ancient Egyptians performed arithmetic.

There is no explicit zero digit in the number system. There are separate characters denoting ones, tens, hundreds, thousands, etc. For the purposes of this problem, we use ASCII symbols for the hieroglyphs: ‘|’ for one; ‘n’ for ten; ‘9’ for hundred; ‘8’ for thousand; ‘r’ for ten-thousand. For this problem, no number greater than 99999 will be used, though the Egyptians were able to express numbers larger than this.

Numbers were written as a group of ones followed in turn by groups of tens, hundreds, thousands, and ten-thousands. Thus 4023 would be represented as ‘||| nn 8888’. Notice that the zero digit in the hundreds place is represented by simply omitting a group of ‘9’s. Similarly, ‘40230’ would be represented as ‘nnn 99 rrrr’. The Egyptians had no way to represent the number zero, but, for the purposes of this problem, zero is represented by the empty string.

To multiply two numbers, *a* and *b*, the Egyptians would work with two columns of numbers. They would begin by writing the number ‘|’ in the left column beside the multiplicand *a* in the right column. They would proceed to form new rows by doubling the numbers in both columns. (Doubling was accomplished simply by doubling the number of each symbol and replacing groups of ten with the next larger symbol.) Doubling would continue as long as the number in the left column did not exceed the multiplier *b*. The numbers in the first column that happen to add up to *b* (and there is only one way to make this sum) were marked with an asterisk. The numbers in the right column alongside the asterisks were then added to produce the result.

Here is how the Egyptians would multiply 483 by 27:

```

* |                               ||| nnnnnnnn 9999
* ||                             ||||| nnnnnn 999999999
  |||                             || nnn 999999999 8
* |||||                          ||| nnnnnn 99999999 888
* ||||| n                         ||||| nn 9999999 8888888
The solution is: | nnnn 888 r
    
```

The left column corresponds to 1, 2, 4, 8, 16. $1+2+8+16=27$, so these rows are indicated with an asterisk. The right column corresponds to 483, 966, 1932, 3864, 7728. The numbers in the right columns in the starred rows are added: $483+966+3864+7728=13041$, which is the correct answer.

You are to write a program to emulate Egyptian multiplication.

INPUT SPECIFICATION

You will be given a series of test cases followed by END<EOLN>. Each test case consists of two Egyptian numbers, *a* followed by *b*: exactly one space will fall between groups of digits; no leading or trailing spaces will appear; each number will be followed by <EOLN>. There will be no extra spaces or <EOLN>'s in the file.

OUTPUT SPECIFICATION

The cases in the output should appear in the same order that they do in the input. Each case should appear as in the above example: The asterisk, if it exists, should appear in column 1 of the line. The left number should begin at column 3. The right number should begin at column 53. Exactly one space should appear between groups of numbers. No additional leading or trailing spaces should appear other than those required to print the numbers in the appropriate column in the output. <EOLN> should follow each row. After the last row is printed, an additional row should be printed, containing the following: “The solution is: ” followed by the Egyptian number—formatted

similarly—corresponding to the final product, followed by <EOLN>. An extra <EOLN> should follow each test case.

SAMPLE INPUT

```

||<EOLN>
||<EOLN>
|||<EOLN>
||||<EOLN>
nnnnnn 9<EOLN>
||| n<EOLN>
n<EOLN>
9<EOLN>
|||<EOLN>
8<EOLN>
<EOLN>
n<EOLN>
END<EOLN>
<EOF>

```

SAMPLE OUTPUT

```

|
* ||
The solution is: |||<EOLN>
<EOLN>
|
||
* ||||
The solution is: || n<EOLN>
<EOLN>
* |
|
* ||||
* |||||
The solution is: nnnnnnn 88<EOLN>
<EOLN>
|
||
* ||||
|||
||| n
* || nnn
* ||| nnnnn
The solution is: 8<EOLN>
<EOLN>
|
||
|||
* |||||
||| n
* || nnn
* ||| nnnnn
* ||||| nn 9
* ||||| nnnnn 99
* || n 99999
The solution is: 888<EOLN>
<EOLN>
|
* ||
|||
* |||||
The solution is: <EOLN>
<EOLN>
<EOF>

```

```

||<EOLN>
||||<EOLN>

|||<EOLN>
|||||<EOLN>
|| n<EOLN>

nnnnnn 9<EOLN>
nn 999<EOLN>
nnnn 999999<EOLN>
nnnnnnnn 99 8<EOLN>

n<EOLN>
nn<EOLN>
nnnn<EOLN>
nnnnnnnn<EOLN>
nnnnnn 9<EOLN>
nn 999<EOLN>
nnnn 999999<EOLN>

|||<EOLN>
|||||<EOLN>
|| n<EOLN>
|||| nn<EOLN>
||||| nnnn<EOLN>
||||| nnnnnnnnn<EOLN>
|| nnnnnnnnn 9<EOLN>
||| nnnnnnnn 999<EOLN>
||||| nnnnnn 99999999<EOLN>
||||| nnn 99999 8<EOLN>

<EOLN>
<EOLN>
<EOLN>
<EOLN>

```