

Problem 1: Making Waves

Suppose we know a signal is generated by the function $x = \sin(f_1 t) + \sin(f_2 t)$. f_1 and f_2 are two unique frequencies, each having an integral value in the range 400 to 600 Hz (Hz = cycles per second), and t represents time. In this problem, you will be given N samples of this function at equally-spaced time intervals corresponding to $t = 1/N$, $t = 2/N$, and so forth. From these samples, you are to determine f_1 and f_2 .

For example, suppose $f_1 = 400$ Hz, $f_2 = 500$ Hz, and $N = 100$. The first sample, at time $t = 1/100$ sec, is equal to $\sin(400 \cdot 0.01) + \sin(500 \cdot 0.01) = \sin(4) + \sin(5) = 0.156912$. Similarly, the second sample, at time $t = 2/100$ sec, is equal to $\sin(400 \cdot 0.02) + \sin(500 \cdot 0.02) = \sin(8) + \sin(10) = 0.312821$.

Input

There will be multiple cases to consider. Each case begins with an integer N , no larger than 1000, that specifies the number of signal samples. The next N data items are real numbers representing the signal samples at time $1/N$ sec, $2/N$ sec, and so forth. A single integer 0 follows the last case.

The number of samples for each case is guaranteed to be sufficient to allow the correct result to be obtained.

Output

For each input case, display a single line that is formatted like this:

```
Case 1, f1 = 400, f2 = 500
```

Sample Input

```
100
0.156912 0.312821 0.466731 0.617657 0.764638 0.906737 1.04305 1.17271
1.29489 1.40883 1.51381 1.60917 1.69432 1.76873 1.83195 1.8836 1.92338
1.95106 1.96649 1.96962 1.96045 1.93908 1.9057 1.86055 1.80396 1.73634
1.65816 1.56997 1.47237 1.36603 1.25166 1.13003 1.00196 0.868307 0.729943
0.587785 0.442764 0.295823 0.147918 1.68756e-010 -0.146981 -0.292088
-0.434403 -0.573031 -0.707107 -0.835801 -0.958325 -1.07394 -1.18195
-1.28171 -1.37266 -1.45428 -1.52611 -1.58779 -1.63898 -1.67947 -1.70907
-1.7277 -1.73535 -1.73205 -1.71795 -1.69323 -1.65816 -1.61308 -1.55838
-1.49452 -1.42201 -1.34141 -1.25334 -1.15846 -1.05745 -0.951057 -0.840028
-0.725146 -0.607206 -0.487017 -0.365392 -0.243145 -0.121082 -2.75143e-010
0.119322 0.236125 0.34968 0.459289 0.564288 0.664055 0.758014 0.845635
0.926438 1 1.06595 1.12398 1.17384 1.21533 1.24833 1.27276 1.28862
1.29596 1.29489 1.28558
```

```
0
```

Expected Output

```
Case 1, f1 = 400, f2 = 500
```